Closing Thur: 12.6, 13.1
Closing *next* Tues: 13.2, 13.3
Closing *next* Thur: 13.4
Exam 1 is next Thurs (April 19)
covers 12.1-12.5, 13.1-13.4

13.1: Intro to 3D Vector Curves

To visualize 3D-curves, we start by
Step 1: Find surface/path of motion.
Step 2: Plot points.

2D Examples

Eliminate the parameters

1. $x = t, y = 2 - t^2$

2. $x = 3\cos(4t), y = 4\sin(4t)$

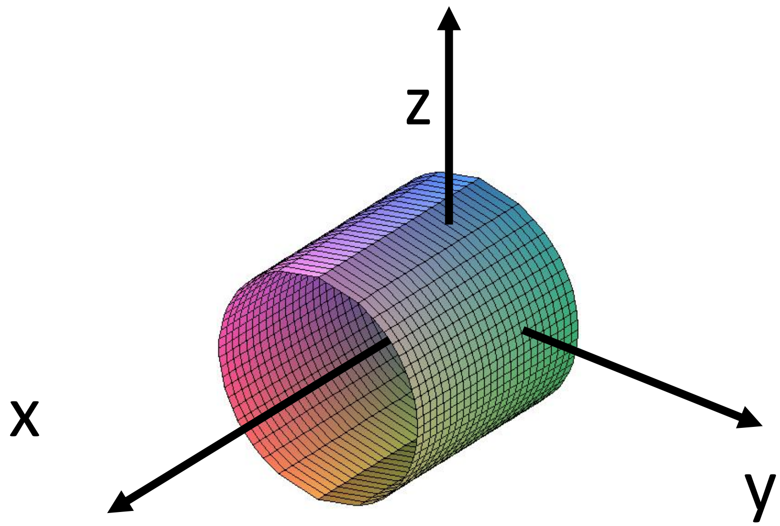
3D Example

$x = t, y = \cos(2t), z = \sin(2t)$

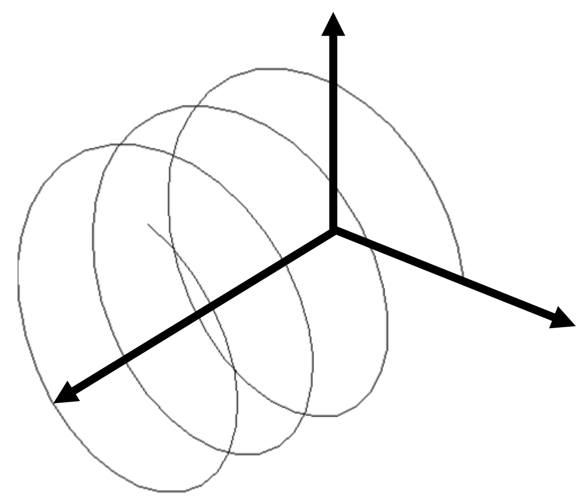
Example: All pts given by the equations

$$x = t\,, y = \cos(2t)\ , z = \sin(2t)$$

are on the cylinder*:* $y^2 + z^2 = 1.$



Now plot points!



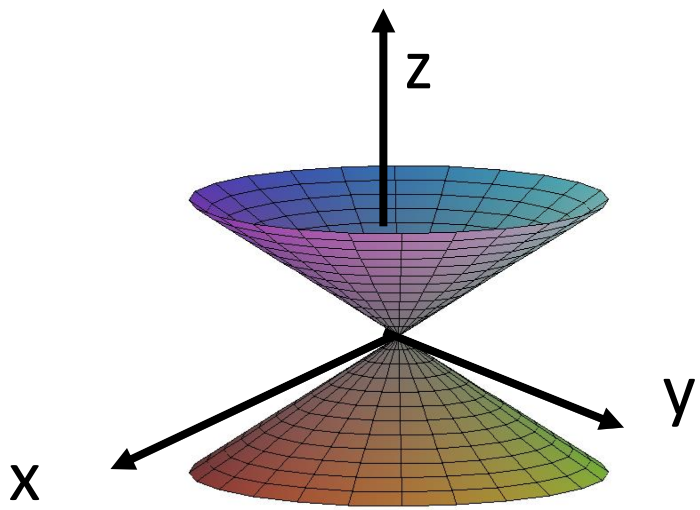
Another 3D Examples

$$x = t\cos(t), y = t\sin(t)\ , z = t$$

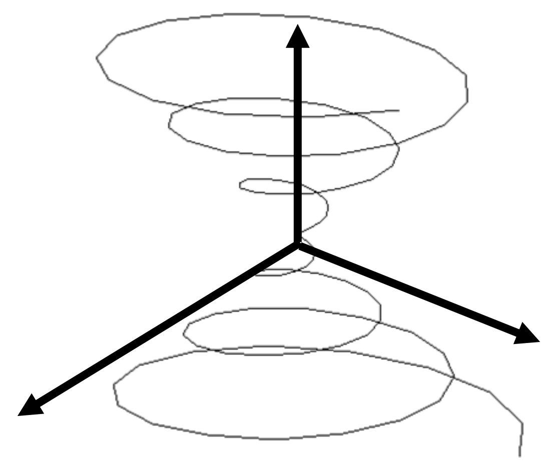
Example: All pts given by the equations

$$x = t\cos(t), y = t\sin(t)\,, z = t$$

are on the cone $z^2 = x^2 + y^2$.

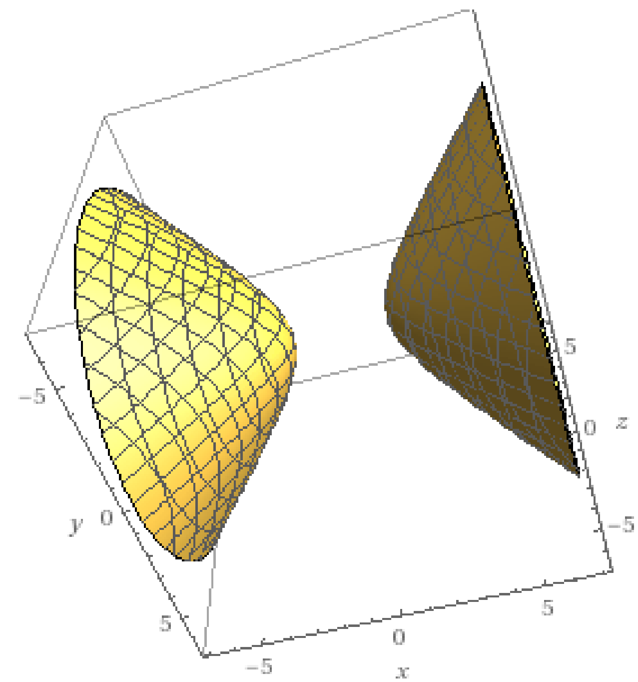


Now plot points!



<u>Intersection issues</u>

For all intersection questions, combine the conditions



(a) ***Intersecting a curve and surface***.

Combine conditions

Example:

Find all intersections of

$$x = t, y = \cos(\pi\, t)\,, z = \sin(\pi\, t)$$

with the surface

$$x^2 - \ y^2 - z^2 = 3.$$

(b) ***Intersecting two curves***.

Use two different parameters!!!

Combine conditions.

We say the objects **collide** if the intersection happens at the same parameter value (i.e. same time).

Example:

Two particles are moving according to

$$\boldsymbol{r}_1(t) = \langle t, 5t, 9\rangle, \text{and}$$
$$\boldsymbol{r}_2(t) = \langle t-2, 5, t^2\rangle.$$

Do their paths intersect?

Do they collide?

(c) ***Intersecting two surfaces***.

Answer will be a 3D curve.

To parameterize the curve:

Let one variable be t. Solve for others in terms of t.

OR

For circle/ellipse try

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \Leftrightarrow \begin{array}{l} x = \mathrm{a}\cos(t) \\ y = \mathrm{b}\sin(t) \end{array}$$

Examples

1. Find *any* parametric equations that describe the curve of intersection of

$$z = 2x + y^2 \text{ and } z = 2y$$

2. Find *any* parametric equations that describe the curve of intersection of
$$x^2 + y^2 = 1 \text{ and } z = 5 - x$$

3. Find *any* parametric equations that describe the curves of intersection of
$$x^2 + y^2 + z^2 = 1 \text{ and } z^2 = x^2 + y^2$$